# Contribution à l'analyse de données non vectorielles Nathalie Villa-Vialaneix

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#### Overview of the topics

data: non vectorial data, high dimension... methods: machine learning & data mining, kernel methods, neuronal methods...





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#### Overview of the topics



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# Functional Data Analysis





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# FDA framework and examples

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Observations of a random variable X taking values in a space of functions \mathcal{X} (typically, an infinite dimensional Hilbert space as L^2): x_1, \ldots, x_n.
The function is observed at t_{i1}, \ldots, t_{id_i}.
```



Applications in: time series analysis, speech recognition, biochemistry (NIR spectra for instance), metabolomic data (NMR), weather data...





## Typical issues in FDA

variance operator used in standard model does not have a continuous inverse: Γ<sub>X</sub> = E(X ⊗ X) - E(X) ⊗ E(X) is a Hilbert Schmidt operator ⇒ Γ<sub>X</sub><sup>-1</sup> is not bounded.
 As a consequence, Γ<sub>X</sub><sup>n</sup> = 1/n Σ<sub>i</sub> x<sub>i</sub> ⊗ x<sub>i</sub> - x̄ ⊗ x̄ is ill-conditionned (and its inverse a bad estimate of Γ<sub>X</sub><sup>-1</sup>).
 ⇒ regularization or penalization techniques are needed.





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 ⇒ regularization or penalization techniques are needed.

(x<sub>i</sub>)<sub>i</sub> are never perfectly observed and only a digitized (sometimes noisy) estimation is available.

 $\Rightarrow$  reconstruction techniques are needed to provide a functional representation of the data, remove noise and measurement artefacts (translation, scaling, ..., of the functions).





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#### Supervised learning framework

(X, Y) st  $X \in X$ ,  $Y \in \mathbb{R}$  (regression) or  $Y \in \{-1, 1\}$  (binary classification). Observations:  $(x_i, y_i)_{i=1,...,n}$ **Purpose**: estimate  $\hat{y}$  for a new x.  $(x_i, y_i)_i$  is used to define a prediction function  $\Phi^n$ .





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Furpose: estimate y for a new x.  $(x_i, y_i)_i$  is used to define a prediction function  $\Phi^n$ .

- Inverse methods use  $\mathcal{L}(X|Y)$  to estimate  $\mathcal{L}(Y|X)$ 
  - [2] (Ferré & Villa, Scandinavian Journal of Statistics, 2006) in the FIR (Functional Inverse Regression) model

$$Y = F(\langle X, \beta_1 \rangle, \ldots, \langle X, \beta_d \rangle, \epsilon),$$

with *F* and  $(\beta_j)_j$  to be estimated, smooth estimation of the  $(\beta_j)_j$  combined with the estimation of *F* by multi-layer perceptron.





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  - ▶ [2] (Ferré & Villa, Scandinavian Journal of Statistics , 2006)
  - [3] (Hernández, et al., Statistica Sinica, 2014) calibration problem in chemiometry: under the assumption that  $\mathcal{L}(X|Y)$  is Gaussian, estimation of

$$f(x|y) = \exp\left[\sum_{j\geq 1} \frac{r_j(y)}{\lambda_j} \left(x_j - \frac{r_j(y)}{2}\right)\right]$$

which is used in a plug-in estimate of  $\mathbb{E}(Y|X = x)$ .





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- Inverse methods use  $\mathcal{L}(X|Y)$  to estimate  $\mathcal{L}(Y|X)$ 
  - ▶ [2] (Ferré & Villa, Scandinavian Journal of Statistics , 2006)
  - ▶ [3] (Hernández, et al., Statistica Sinica, 2014)
- Kernel methods (less sensitive to high dimension, can include some functional pre-processing)
  - ▶ [6] (Rossi & Villa, Neurocomputing, 2006) SVM for functional data analysis
  - [8] (Rossi & Villa-Vialaneix, Pattern Recognition Letter, 2011) derivative-based kernel methods for functional classification and regression



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  - ▶ [6] (Rossi & Villa, Neurocomputing, 2006)
  - ▶ [8] (Rossi & Villa-Vialaneix, Pattern Recognition Letter, 2011)
- [5, 12], (Rohart, et al., Journal of Animal Science, 2012), (Villa-Vialaneix, et al., Communication in Statistics, 2014) biomarker identification from metabolomic (NMR) data using functional approaches

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# Kernels for functional data

Given  $K : \mathcal{X} \times \mathcal{X} \to \mathbb{R}$  st

• symmetry: K(x, x') = K(x', x)

• positivity:  $\forall N \in \mathbb{N}, \forall (\alpha_i) \subset \mathbb{R}^N, \forall (x_i) \subset \mathcal{X}^N, \sum_{i,j} \alpha_i \alpha_j K(x_i, x_j) \ge 0.$ 

 $\exists! (\mathcal{H}, \langle ., . \rangle_{\mathcal{H}}) \text{ (RKHS) and } \Psi : \mathcal{X} \to \mathcal{H} \text{ st } \mathcal{K}(x, x') = \langle \Psi(x), \Psi(x') \rangle_{\mathcal{H}}$ 

#### General form for FD

pre-processing:  $\mathcal{P} : \mathcal{X} \to \mathcal{D}$ 

$$\forall x, x' \in \mathcal{X}, Q(x, x') = K(\mathcal{P}(x), \mathcal{P}(x')).$$

• projections: for  $V_d = \text{Vect} \{\psi_1, \dots, \psi_d\}, \mathcal{P}(x) = \sum_{j=1}^d \langle x, \psi_j \rangle \psi_j$  (and  $K = K_d$ , standard kernel on  $\mathbb{R}^d$ ).

- In the functional transformation:  $\mathcal{P}(x) = D^q x, \dots$
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# A consistent approach

[6] (Rossi & Villa, *Neurocomputing*, 2006) SVM for functional data:  $(X, Y) \in X \times \{-1, 1\}$  with kernel  $Q = K_d \circ \mathcal{P}_{V_d}, K_d$  standard kernel on  $\mathbb{R}^d$ 



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## Assumptions

[6] (Rossi & Villa, Neurocomputing, 2006)

#### Assumptions on the law of X

(A1) X takes its values in a bounded subspace of X.

#### Assumptions on the parameters: $\forall d \ge 1$ ,

(A2)  $\mathcal{J}_d$  is a finite set; (A3)  $\exists \mathcal{K}_d \in \mathcal{J}_d$  (kernels on  $\mathbb{R}^d$ ) st:  $\mathcal{K}_d$  is universal and  $\exists \nu_d > 0 : \mathcal{N}(\mathcal{K}_d, \epsilon) = O(\epsilon^{-\nu_d});$ (A4)  $C_d > 1$ ; (A5)  $\sum_{d \ge 1} |\mathcal{J}_d| e^{-2\lambda_d^2} < +\infty.$ 

#### Assumptions on the training/validation sets

(A6) 
$$\lim_{n \to +\infty} I = +\infty$$
;  
(A7)  $\lim_{n \to +\infty} n - I = +\infty$ ;  
(A8)  $\lim_{n \to +\infty} \frac{I\log(n-I)}{n-I} = 0$ .





## Convergence to the Bayes error

[6] (Rossi & Villa, Neurocomputing, 2006)

#### Theorem: universal consistency

Under the assumptions (A1)-(A8),

$$L\Phi^n \xrightarrow{n \to +\infty} L^*,$$

where

• 
$$L\Phi^n = \mathbb{P}(\Phi^n(X) \neq Y)$$
  
•  $L^* = \mathbb{P}(\Phi^*(X) \neq Y)$  with  $\Phi^*(x) = \begin{cases} 1 & \text{if } \mathbb{P}(Y = 1|X = x) > 1/2, \\ -1 & \text{otherwise.} \end{cases}$ 





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[8] (Rossi & Villa-Vialaneix, Pattern Recognition Letter, 2011) alternative kernel for:

- digitized observations
- derivation preprocessing

with a general consistency result in regression and binary classification.









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## Framework of this section



A graph (network)  $\mathcal{G} = (V, E, W)$  with

- *n* vertices  $V = \{x_1, ..., x_n\}$ ;
- a set of edges, *E*, weighted by  $W_{ij} = W_{ji} \ge 0$  ( $W_{ii} = 0$ ).



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# Visualization: a tool for graph mining

Standard approach for graph visualization: force directed placement algorithms (FDP). Drawbacks:

- slow (impracticable for large graphs);
- based on aesthetic criteria rather than on interpretability:
  - trend: short and uniform length edges;
  - negative consequence: nodes with the largest degrees are grouped in the middle of the layout.





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- slow (impracticable for large graphs);
- based on aesthetic criteria rather than on interpretability:
  - trend: short and uniform length edges;
  - negative consequence: nodes with the largest degrees are grouped in the middle of the layout.
- A more natural way to explore a graph:
  - highlight the macroscopic structure: find "communities" and relations between them;
  - eventually focus on finer details in some communities.





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- topographic maps: combine clustering and visualization using a prior structure (*ie* a map):
  - batch kernel SOM [1] (Boulet, et al., Neurocomputing, 2008) and on-line multiple relational SOM [4] (Olteanu & Villa-Vialaneix, Neurocomputing, 2015) (not restricted to graphs; can handle multiple graphs or labeled graphs)





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  - maps based on an organized modularity criterion [7] (Rossi & Villa-Vialaneix, *Neurocomputing*, 2010)





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- [9] (Rossi & Villa-Vialaneix, *Journal de la SFdS*, 2011) hierarchical approach (based on modularity maximization) for graph clustering and visualization;





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- applications
  - for mining a network extracted from a corpus of medieval documents [10] (Rossi, et al., Digital Medievalist, 2013)





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  - for mining a network extracted from a corpus of medieval documents [10] (Rossi, et al., Digital Medievalist, 2013)
  - for co-expression network analysis [13] (Villa-Vialaneix, et al., PLoS ONE, 2013) (clustering in co-expression network with ontology validation)





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# Using topographic maps to visualize a clustered graph





- vertices are clustered in U units
- units are arranged on a prior 2D-grid equipped with a topology
- prior positions of the units are used to produce a simplified representation





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# How to extend topographic maps to graph?

Generic approach using kernel/dissimilarity

[4] (Olteanu & Villa-Vialaneix, *Neurocomputing*, 2015), [1] (Boulet, *et al.*, *Neurocomputing*, 2008) graphs can be described by pairwise relations between nodes:

- with a kernel (ex:  $K^{\beta} = e^{-\beta L}$ , with *L* the graph Laplacian, heat kernel)
- with a dissimilarity (ex: shortest path length)

⇒ extension of SOM to kernel/dissimilarity data using prototypes of the form  $p_u = \sum_{i=1}^n \gamma_{ui} x_i$ .





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 $\Rightarrow$  extension of SOM to kernel/dissimilarity data using prototypes of the form  $p_u = \sum_{i=1}^n \gamma_{ui} x_i$ .

#### Graph specific approach

[7] (Rossi & Villa-Vialaneix, *Neurocomputing*, 2010) Extension of the modularity quality criterion to a criterion taking into account the prior topology

$$O = \frac{1}{2m} \sum_{ijk} M_{ik} \mathbf{S}_{kl} M_{jk} B_{ij}$$

 $B = (W_{ij} - \frac{d_i d_j}{2m})$  modularity matrix ( $d_i = \sum_j W_{ij}$  and  $m = \frac{\sum_i d_i}{2}$ );  $M_{ik} = \mathbf{1}_{x_i \in k}$ ; S encodes the topology of the prior map (similarity)



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## Practical aspects of kernel/relational SOM

- this approach (and others) is implemented in the R package SOMbrero provided with a WUI based on shiny;
- it can also handle data described by multiple dissimilarities: the combination is optimized by including a gradient descent-like step in the relational SOM algorithm;





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- it can also handle data described by multiple dissimilarities: the combination is optimized by including a gradient descent-like step in the relational SOM algorithm;
- it has been applied to graphs, labeled graphs and to other non vectorial data (school-to-work trajectories)















#### Network inference

Data: large scale gene expression data

individuals  

$$n \simeq 30/50$$

$$\underbrace{\begin{cases} X = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & X_i^j & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}}_{\text{variables (genes expression). } p \simeq 10^4}$$

What we want to obtain: a graph/network with

- nodes: (selected) genes;
- edges: strong links between gene expressions.



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 application of network inference for understanding the determinant of human adipose tissue gene expression [11] (Viguerie, *et al.*, *PLoS Genetics*, 2012)

• network inference with multiple samples [14] (Villa-Vialaneix, et al., Quality Technology and Quantitative Management, 2014)





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# Motivation for multiple networks inference

Pan-European project Diogenes<sup>1</sup> (with Nathalie Viguerie, INSERM): gene expressions (lipid tissues) from 204 obese women before and after a low-calorie diet (LCD).



- Assumption: A common functioning exists regardless the condition;
- Which genes are linked independently from/depending on the condition?

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# Consensus LASSO

#### GGM with $L^1$ penalty: one condition

 $(X_i)_{i=1,\dots,n}$  are i.i.d. Gaussian random variables  $\mathcal{N}(0, \Sigma)$ 

$$X^{j} = \beta_{j}^{\mathsf{T}} X^{-j} + \epsilon$$
 ; arg min  $\sum_{(\beta_{jj'})_{j'}}^{n} \sum_{i=1}^{n} \left( X_{ij} - \beta_{j}^{\mathsf{T}} X_{i}^{-j} \right)^{2} + \lambda ||\beta_{j}||_{L^{1}}$ 

 $j \longleftrightarrow j'$ (genes j and j' are linked)  $\Leftrightarrow \beta_{jj'} \neq 0$ 

#### Consensus LASSO: multiple conditions

$$\frac{1}{2}\beta_j^T \widehat{\Sigma}_{\backslash j \backslash j}\beta_j + \beta_j^T \widehat{\Sigma}_{j \backslash j} + \lambda \|\beta_j\|_{L^1} + \mu \sum_c w_c \|\beta_j^c - \beta_j^{\mathsf{cons}}\|_{L^2}^2$$

with  $\widehat{\Sigma}_{\langle j \rangle j}$ : block diagonal matrix  $\mathbb{D}$ iag  $(\widehat{\Sigma}_{\langle j \rangle j}^{1}, \dots, \widehat{\Sigma}_{\langle j \rangle j}^{k})$  and similarly for  $\widehat{\Sigma}_{j \rangle j}$ :

- w<sub>c</sub>: real number used to weight the conditions;
- $\mu$  regularization parameter;
- $\beta_i^{\text{cons}}$  whatever you want...?



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# Choice of a consensus

#### Case 1: a priori consensus

Using a fixed  $\beta_j^{\text{CONS}}$ , the optimization problem is equivalent to minimizing the *p* following standard quadratic problem in  $\mathbb{R}^{k(p-1)}$  with  $L_1$ -penalty:

$$\frac{1}{2}\beta_j^T B^1(\mu)\beta_j + \beta_j^T B^2(\mu) + \lambda ||\beta_j||_{L^1},$$

#### Case 2: learn the consensus

Using  $\beta_j^{\text{cons}} = \sum_{c=1}^k \frac{n_c}{n} \beta_j^c$ , the optimization problem is equivalent to minimizing the following standard quadratic problem with  $L_1$ -penalty:

$$\frac{1}{2}\beta_{j}^{T}S_{j}(\mu)\beta_{j}+\beta_{j}^{T}\widehat{\Sigma}_{j\setminus j}+\lambda||\beta_{j}||_{L^{1}}$$

Optimization by active set, combined with bootstrap approach (BOLASSO type). R package **therese**.











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# Research project: data mining and integration with SOM

#### On-going issues

- stabilize results, improve quality, speed-up training: aggregation, boosting...
- improve interpretability in a multi-kernel/dissimilarity context: visualization, prototype sparsity...
- targeted applications: multi-'omics integration and exploration; ncRNA typology

#### Collaborations

- methodological aspects: Jérôme Mariette (PhD, MIAT, INRA), Madalina Olteanu (SAMM, Université Paris 1)
- application to multi-'omics data: Nathalie Viguerie (INSERM, Diogenes project)
- application to ncRNA: Christine Gaspin (MIAT, INRA)





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Research project: data mining and integration using graphical approaches

#### On-going issues

- integrating multi-'omics data in network inference and mining, possibly with different numbers of observations
- taking temporal aspects into account in network inference/clustering

#### Collaborations

- methodological aspects: Valérie Sautron (PhD, GenPhySE, INRA)
- applications: projects SusOStress & PigHeat (GenPhySE, INRA) on systems genetics in pigs (stress & heat resistance)
- application: Nathalie Viguerie (INSERM, Diogenes project) & Ignacio Gonzáles (MIAT, INRA)... submitted article





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#### Thank you for your attention...



#### ... questions?





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